Review of Maple 7

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The use of the MAPLE Computer Algebra System is common in Higher Education Institutes in the UK. Reviews of previous versions of the software have appeared in this newsletter and its predecessors, as well as elsewhere [1], [2], [3], [4]. At the University of Plymouth, MAPLE is used in both service teaching (albeit to a limited extent) and on our Mathematics and Statistics degrees. In 1999, we obtained a University-wide site licence under a CHEST agreement. We also decided to subscribe to the 'Campus VIP' programme which, for around £1500 per annum, allows us to distribute the system to our students (we charge each student a nominal sum for the privilege). The system is used on a Windows NT network (for students) and some staff have Linux or Unix setups.

In the first year of our Mathematics degrees, we have a pair of modules newly dedicated to the use of MAPLE. A separate module also introduces MAPLE and various Statistics packages to a mixed first year group. David McMullan uses MAPLE in a first-year course on Mathematical Reasoning, in which the students are eventually able to use it to encrypt and decrypt messages. Other first-year modules make use of DERIVE and various other Statistics packages. In the first MAPLE module, David Wilton introduces the package to our students, most of whom have never used such a system. In this, he draws upon material developed at Bristol University. In David's second module, students are introduced to numerical methods and use MAPLE's own Fortran-like programming language. The response of the students to the use of MAPLE has been much more positive than when Fortran was used in the previous incarnation of the module.

In the second year of our Mathematics degrees, I run a pair of modules which use MAPLE to further explore numerical methods, more of which below. Last year, the module received a mixed reception. From the lecturer's point of view, the integration of programmable symbolics and numerics, together with good graphics and reasonable typesetting capabilities is a strong motivation for using the system. The biggest drawback from the students' point of view was that they were in a transition year and had had no previous exposure to MAPLE. Even though the MAPLE programming language is very similar to the Fortran that they had seen, many students found it difficult to get to grips with the system without formal tutoring.

Colin Christopher uses the extensive graphical capabilities to explore ideas in Advanced Calculus. Some of Colin's MAPLE-generated VRML support material is available at his web site http://www.tech.plym.ac.uk/maths/staff/cchristopher/home.html. The general aim is for our students to be able to use Maple as an exploratory tool. However, expectation and reality have not yet met – perhaps the curriculum does not allow students enough time for the necessary reflection and exploration.

New Features in MAPLE 7

The Autumn 2001 Maple Reporter [5] notes improvements in two main areas: Mathematical Power, and Connectivity with the Web. In the first area, MAPLE now claims to be the undisputed leader in Differential Equations and advertises Units and Dimensional Management and Enhanced Efficiency as major improvements. Connectivity is improved via use of TCP/IP sockets and XML and MATHML support, as well as easier linking to external C code. The resources in the MAPLE Application centre and MAPLE Power Tools are lauded. An additional resource not mentioned in this article, but mentioned

elsewhere in the Maple Reporter is the new Maple4Students site (see below).

Although the list of new features and packages introduced in MAPLE 7 is extensive, many of these will be of little immediate impact in Teaching and Learning (this is true for Plymouth at least). One of the most useful is the **RealDomain** package, which forces many commonly-used functions and procedures such as trigonometric and hyperbolic functions, **simplify** and **limit** to work under the assumption that their arguments are real. Along similar lines, the new **assuming** facility is a useful addition and also works within the ODE and PDE solvers **dsolve** and **pdsolve**.

There are also some welcome changes are to the treatment of both ordinary and partial differential equations. Both symbolics and numerics have improved – the procedures are more powerful, efficient and flexible. For example, the user is now able to specify stopping criteria (for example to stop the computation if a variable becomes negative) and to force arbitrary-precision numerical computations of ODE's and PDE's. The participation of NAG has also increased – they now have input to the numerical quadrature schemes as well as further involvement with numerical linear algebra.

The **Sockets** package is extremely interesting and potentially very useful. This enables MAPLE users to interrogate WWW pages for information. The web page at http://www.adeptscience.co.uk/go?pg=H31 gives a lovely example in which MAPLE also uses the new text string processing package **StringTools** to find the current temperature at 30 cities around the world from http://weather.lycos.com. MAPLE then plots the data as colour-coded points at the cities' locations on the globe.

The new **Units** package should prove to be useful in teaching science and engineering courses. The system knows of 52 dimensional quantities (from **absorbed [radiation] dose** to **work or energy**), each of which has conversion factors from one system to another. As an example, there are 15 different units of angular measurement, from **angle hours** to **signs** and the system knows of the conversion factors to translate between units.

The **MATHML** package will eventually prove to be useful in porting MAPLE mathematics to the WWW. At the moment, however, with the MATHML-enablement of web browsers being in its infancy, viewing of the equations using something like WebEq is time-consuming. This should improve in the short to medium term when MATHML is fully incorporated into Netscape and Internet Explorer.

On the downside, some of the documentation is out of sync with the code developments, as previous reviewers have mentioned. For example, plot legends were available in MAPLE 6 but were undocumented. This has been rectified in MAPLE 7 (at least on the online system, if not in the paper guides). The **simplify** command can now be used with a **size** argument to simplify some algebraic expressions. This was also present in MAPLE 6 but undocumented. In addition, it does not appear on the MAPLE 7 help system, but only in the list of new features under "Changes to Maple's symbolic capabilities in various packages and procedures. See updates/Maple7/symbolic". Waterloo should put some effort into bringing the documentation and help system up to date. Whilst we're on the subject of **simplify**, I welcome anything that really does make expressions more readable or concise but wonder about the following example, which shows that the **simplify** command continues to be problematical:

```
> simplify(x*y+x*z+h*y+h*j, size);
```

$$(y+j)h+(y+z)x$$

```
> simplify(x*y+x*z+h*y+h*j+1, size);
```

$$(y+j)h+xy+xz+1$$

The only changes to the appearance are the useful **Restart** and **Execute Worksheet** buttons added to the Worksheet Toolbar and Context Bar. In addition, a new vector palette is available, in addition to the **Symbol**, **Expression** and **Matrix** palettes available in MAPLE 6.

It is worth noting that almost all of the bugs referred to in reviews of previous versions of MAPLE now appear to have been overcome. In addition, apparent bugs that I had found in the **limit** and **int** procedures have now been fixed.

Documentation

The documentation has changed slightly from release 6, with the Learning and Programming Guides now being complemented by a handy Getting Started Guide (although this is not a great advertisement for the document-processing capabilities of MAPLE, having a very ugly mixture of text sizes). This gives a good first exposure to MAPLE 7. The Learning Guide has been rewritten in sympathy. In addition to the guides, the online **Maple Essentials** guides provide basic sessions on numerical and symbolic calculations, graphs, solving equations and functions, including a good supply of practice problems: see http://www.mapleapps.com/categories/whatsnew/html/SCCCmapletutorial.html.

Maple Resources

Maple4Students, a new site dedicated to students' use of MAPLE has recently appeared at http://www.Maple4Students.co.uk. This has some very useful resources for both lecturer and student, including links to the **Maple Essentials** tutorials mentioned above. At the moment there are only a few examples of work submitted by students. Generally, the results are quite impressive – there are some nice animations and graphic examples, including a lovely animation of a dripping tap.

The place of the **share** library once distributed with MAPLE is now taken by the **Maple Application Centre** at http://www.adeptscience.co.uk/products/mathsim/maple/apps/. It contains many examples of worksheets generated by users around the world looking at more or less realistic problems. Unfortunately, many of the examples that I tried didn't work – some used obsolete packages (eg ODE). Lots of these examples are extremely interesting real problems and are certainly worth updating. The **Maple PowerTools** at http://www.adeptscience.co.uk/products/mathsim/maple/powertools/ provide more carefully quality-controlled and better-documented resources covering topics including fairly extensive courses on Post-Secondary Mathematics, Calculus (three courses), Differential Equations and Complex Analysis.

Writing Documents in MAPLE

Many of the examples in Maple4Students and in the Application Centre are very interesting, well designed and effective. However, it must be said that MAPLE's text-processing facilities remain a little basic. Even though proper mathematical notation can be used, it is difficult to produce documents of great beauty with the system. Online MAPLE worksheets themselves can be made to look very impressive, especially with the use of sections, paragraphs and hyperlinks. Using hyperlinks, it is possible to define links to other location in a worksheet, or to another worksheet, to the help system or even to WWW pages. This facility, introduced in MAPLE 6, is excellent. These links are preserved when exporting output to HTML. However, the output, with or without MATHML, is a poor second best to viewing within MAPLE itself.

Programming in MAPLE

Much of my second-year course on Computational Mathematics is devoted to the numerical solution of initial-value problems. A great feature of systems such as MAPLE is the ability to solve the IVP both numerically

and analytically (for simple problems, at least) and the graphics mean that it's easy to compare results. One of my favourite examples of the power MAPLE is the ability to specify an arbitrary-order Taylor-series scheme for solving such problems. Suppose that we want to solve the IVP

$$\frac{dy}{dx} = f(x, y), \ y(0) = y_0$$

We specify the mth order Taylor series method using a 20-line procedure:

```
> TaylorIVP:=
proc(f::procedure, x0, y0, h, 2^n, m)
```

whose arguments are the function $f(x, y)$, the starting values x_0 and y_0, the x-increment h, the number of x-intervals 2^n, and the order of the method m. Such a general specification would be impossible without the symbolic algebra capability.

Further simple programming enables us to demonstrate the order of convergence of the methods. The following table summarises some results from this procedure (for the case considered here –
$f(x, y)=-y+x+1$, $x_0=0$, $y_0=1$ — an analytical solution is available for comparison with the numerical results).

The numbers in the first row indicate the number of divisions 2^n. The numbers in the first column show the order of the method used, m. The entries in the body of the table show the ratios of the global error (ie the difference between the analytical and computational results at $x=1$) using 2^n and 2^{n+1} divisions. These ratios should approach 2^m as the number of intervals increases. This behaviour is clearly demonstrated in the table. Reasonably attractive formatted output as below is generated by writing the error ratios into an $m+1$ x $n+1$ matrix and simply printing the matrix – default output is far less pretty.

$$\begin{bmatrix} 1, & 2, & 4, & 8, & 16, & 32 \\ 1\,, & 3.12081086845, & 2.29011332872, & 2.12081075323, & 2.05589890310, & 2.02695788874 \\ 2\,, & 5.80863103068, & 4.89195074993, & 4.41219946515, & 4.19678694595, & 4.09605898513 \\ 3\,, & 12.0702800888, & 9.77870691117, & 8.84451555878, & 8.41121673487, & 8.20278131059 \\ 4\,, & 24.4354333540, & 19.7450924059, & 17.7649714114, & 16.8509127789, & 16.3787375415 \end{bmatrix}$$

Another feature that I am particularly fond of is the ability to specify the arithmetic precision of numerical computations. A recent review [4] has noted that such computations are IEEE 754-compliant and should therefore be reliable. A good example is the use of iterative refinement to improve the solutions to systems of linear equations using finite-precision arithmetic.

Students are given a procedure called **RefineIt**

```
> RefineIt:=proc(A, b, ndim, ndigits,
tol, niter)
```

which solves the system defined by the *ndim* x *ndim* matrix ***A*** and right hand side vector ***b***. All arithmetic is performed using *ndigits* digits, except the computation of residuals which uses *ndigits**2. Iteration continues until the norm of the difference between successive results is ≤ *tol*, up to a maximum of *niter* iterations.

Solving the ill-conditioned system represented by the 2x3 augmented matrix

$$\begin{bmatrix} 3.9 & 1.6 & 5.5 \\ 6.8 & 2.9 & 9.7 \end{bmatrix}$$

using Gaussian elimination with 3-digit arithmetic gives

"Gaussian elimination: x=" , [.863, 1.33]

With iterative refinement, using double precision to compute the residual, the solution becomes exact:

"Iteration" , 3, "x=" , [1.00, 1.00]

Again, by being given a general-purpose procedure able to accept non-singular square matrices of any size, students can explore ill-conditioned systems such as those defined by Hilbert matrices of any given order. They can use arithmetic of any precision to do so, which would be impossible in a conventional programming language. The **RefineIt** procedure is fairly complex and it is debatable whether students unfamiliar with programming would be able to generate for themselves procedures such as these. For more experienced programmers, however, the potential for creating designer procedures and packages is great.

Random List of Other Sources of Irritation

MAPLE is an excellent system for supporting the teaching and learning of mathematics. As noted above, however, there are several aspects which are more or less frustrating. In addition to those negatives mentioned already, I include the following list:

1. The FlexLM licensing system introduced with MAPLE 6 continues to cause problems. This University has to run 'standalone' versions of the software on our Windows NT network. We have had problems with license files running out or simply not working or even belonging to a different University! The distribution of MAPLE 7 was apparently delayed for several weeks due to problems with Unix licensing.

2. The 'stop' button almost never works when you need it to. This was mentioned in the review of MAPLE 4 in 1997 [1] !

3. The system struggles to generate detailed plots. Even on a Unix workstation with 1GByte of RAM, the time taken to generate graphics such as the Mandelbrot set, final-state diagrams and chaotic attractors was excessive.

4. Documentation on 'export'ing graphics is almost non-existent. When exporting Postscript graphics, the default should be colour rather than grayscale. Otherwise, it should be possible to specify options when using this facility.

5. It should be easily possible to make the 'standard math' representation of Diff(f, x) have a straight d/dx, rather than using partial derivatives as at present.

Conclusions

MAPLE continues to improve. For teaching and learning purposes, there are not many significant differences between MAPLEs 6 and 7. Our experience with MAPLE 6 shows that students like it, but should be taught formally how to use it. The potential for use in reinforcing teaching of many aspects of mathematics is great. It is particularly good for numerical analysis, enabling exploration of areas previously unmapped by conventional programming languages. Some glitches and sources of frustration remain, although all of the bugs referred to in previous reviews [1], [2] and [3], seem to have been ironed out.

References

[1] P Whitworth, Review of Maple V Release 4, *Maths&Stats* Vol 8, No 1, Feb 1997

[2] N Backhouse, Review of Maple V Release 5, *Maths&Stats* Vol 9, No 3, Aug 1998

[3] N Backhouse, Review of Maple 6, *Maths Stats & OR* Newsletter #2, May 2000

[4] S S Ross, Maple 6: The Analytical Engine, *NASA Technical Briefs*, July 2000

[5] Waterloo Maple Inc, *The Maple Reporter*, Waterloo Maple Inc, Autumn 2001